Tenta 1 MS005
U. Menzel

1. a) $\sum_k p_X(k) = 1 \rightarrow p_X(4) = 0.1$

b)

k	1	2	3	4
$F_X(k)$	0.2	0.7	0.9	1

c) $P(X<3) = P(X \leq 2) = F_X(2) = 0.7$

d) $E(X) = \sum_k k \cdot p_X(k) = 0.2 + 1 + 0.6 + 0.4 = 2.2$

e) $V(X) = E(X^2) - E(X)^2 = 5.6 - 2.2^2 = 0.87$

$E(X^2) = 1 \cdot 0.2 + 4 \cdot 0.5 + 9 \cdot 0.2 + 16 \cdot 0.1 = 5.6$

2.

	S	S^*	
D	0.05	0.01	0.06
D^*	0.02	0.92	0.94
	0.07	0.93	1

S = signal
D = defekt

a) $P(D) = 0.06$

b) $P(D^*) = 0.94$

c) $P(S^* \mid D) = \frac{P(S^* \cap D)}{P(D)} = \frac{0.01}{0.06} = \frac{1}{6}$

d) $P(S \mid D^*) = \frac{P(S \cap D^*)}{P(D^*)} = \frac{0.02}{0.94} = \frac{1}{47}$

e) $P(\text{fel}) = P(S^* \cap D) + P(S \cap D^*) = 0.01 + 0.02 = 0.03$

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(3) $F_X(t) = \int_{-\infty}^{t} f_X(u)\,du$

$$= \int_0^t \frac{3}{64} u^2(4-u)\,du = \frac{3}{64}\left[4\int_0^t u^2 du - \int_0^t u^3 du\right]$$

$$= \frac{3}{64}\left[4\cdot \frac{u^3}{3}\Big|_0^t - \frac{u^4}{4}\Big|_0^t\right] = \frac{3}{64}\left[\frac{4}{3}t^3 - \frac{1}{4}t^4\right]$$

Check: $F_X(0) = 0$ ✓

$F_X(4) = 1$ ✓

a) $$F_X(t) = \begin{cases} 0 & t \le 0 \\ \frac{1}{16}t^3 - \frac{3}{256}t^4 & 0 < t \le 4 \\ 1 & t > 4 \end{cases}$$

b) $P(X \le 1) = F_X(1) = \frac{1}{16} - \frac{3}{256} = \frac{13}{256} = 5.1\%$

c) $E(X) = \int_{-\infty}^{+\infty} t\cdot f_X(t)\,dt = \frac{3}{64}\int_0^4 t^3(4-t)\,dt = \ldots\ldots$

$$= \frac{3}{64}\left[t^4\Big|_0^4 - \frac{t^5}{5}\Big|_0^4\right] = \frac{12}{5} = 2.4$$

d) kostnader $K = 2000\cdot X$

$E(K) = E(2000\cdot X) = 2000\cdot E(X) = 4800$

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(4) $X_i \in N(200, 0.5)$ $\mu = 200$ $\sigma = 0.5$

$\bar{X} \in N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ $n = 25$

$\bar{X} \in N(200, 0.1)$

$$P(|\bar{X}-\mu| > 0.25) = 1 - P(|\bar{X}-\mu| \leq 0.25)$$

$$= 1 - P(-0.25 < \bar{X} - \mu \leq 0.25) \qquad \mu = 200$$

$$= 1 - \underbrace{P(199.75 < \bar{X} \leq 200.25)}_{\text{sth. att } \bar{X} \text{ inte avviker mer än } 0.25\,!}$$

$$= 1 - F_{\bar{X}}(200.25) + F_{\bar{X}}(199.75)$$

$$= 1 - \Phi\left(\frac{200.25-200}{0.1}\right) + \Phi\left(\frac{199.75-200}{0.1}\right)$$

$$= 1 - \Phi(2.5) + \Phi(-2.5)$$

$$= 1 - \Phi(2.5) + 1 - \Phi(2.5)$$

$$= 2 - 2\cdot\Phi(2.5)$$

$$= 2 - 2\cdot 0.99379$$

$$= 0.0124$$

$$\approx 1.24\%$$

(5) $\frac{\bar{x}-\mu}{s/\sqrt{n}} \in t(n-1)$

$I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}$

$n = 8$
$f = 7$
$\alpha = 0.05$

$\bar{x} = 199.29$

$s = 0.716$

a) $I_\mu = 199.3 \pm 0.6$

$= (198.7\,;\,199.9)$ 95% kI

b) Nej, med 95% konfidens är kullagren för små.

(6) a) $X \in Bin(n, p)$

b) $p_{obs} = \frac{x}{n} = \frac{20}{100} = 20\%$

c) $I_p = p_{obs} \pm \lambda_{\alpha/2} \cdot \sqrt{\frac{p_{obs}(1-p_{obs})}{n}}$ normalapproximation

$= 0.2 \pm 1.96 \cdot \sqrt{\frac{0.2 \cdot 0.8}{100}}$

$= 0.2 \pm 1.96 \cdot 0.04 = 0.2 \pm 0.0784$

$\approx 0.2 \pm 0.08$

d) bredd: $b = 2 \cdot \lambda_{\alpha/2} \cdot \sqrt{\frac{p(1-p)}{n}} = 2 \cdot 0.0784 = 0.1568$

$\leadsto$

$n = p(1-p) \cdot 4 \cdot \left(\frac{\lambda_{\alpha/2}}{b}\right)^2$ med $\alpha = 0.01 \rightarrow \lambda_{\alpha/2} = 2.58$

$n = 173$ om man antar att p blir detsamma: $p = 0.2$

$n_0 = 271$ ... om man inte gör det: $p = \frac{1}{2}$ (maximum)

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(7) $$\frac{\bar{x}_A - \bar{x}_B - \Delta\mu}{s_p\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}} \in t(f) \qquad f = n_A + n_B - 2 = 20$$

$$I_{\Delta\mu} = \bar{x}_A - \bar{x}_B \pm t_{\alpha/2}(f)\cdot s_p \cdot \sqrt{\frac{1}{n_A}+\frac{1}{n_B}}$$

$$s_p^2 = \frac{(n_A-1)s_A^2 + (n_B-1)s_B^2}{f}$$

$$= \frac{9\cdot 5.4^2 + 11\cdot 5.2^2}{20} = \frac{262.44 + 297.44}{20}$$

$$= 28$$

$$s_p = 5.29$$

$$t_{\alpha/2}(f) = t_{0.025}(20) = 2.09$$

a) $$I_{\Delta\mu} = 28.3 - 31.1 \pm 2.09 \cdot 5.29 \cdot \sqrt{\frac{1}{10}+\frac{1}{12}}$$

$$= -2.8 \pm 4.735$$

$$= (-7.53;\ 1.93) \qquad 95\%\ \text{KI}$$

b) Nej, konfidensintervallet innehåller 0.

(8) $z = (3\,;\ 4\,;\ 3\,;\ 5\,;\ 0\,;\ -1\,;\ 3\,;\ 3)$

$$\frac{\bar{z} - \Delta\mu}{s_z/\sqrt{n}} \in t(n-1)$$

$$I_{\Delta\mu} = \bar{z} \pm t_{\alpha/2}(n-1) \cdot \frac{s_z}{\sqrt{n}}$$

$$= 2.5 \pm 2.36 \cdot \frac{2}{\sqrt{8}}$$

$$= 2.5 \pm 1.67$$

$$\approx (0.83\,;\ 4.17)$$

$\bar{z} = 2.5$

$s_z = 2$

$t_{\alpha/2}(n-1) = t_{0.025}(7)$

$= 2.36$