1.) $p_X(k) = c \cdot \frac{2}{3} \cdot k^2$ $\qquad \sum_{k=1}^{4} p_X(k) = 1$ $\qquad$ tenta 23/10-2018

a) $c \cdot \frac{2}{3} \sum_{k=1}^{4} k^2 = 1$

$c \cdot \frac{2}{3}\left(1^2+2^2+3^2+4^2\right) = 1$

$c \cdot \frac{2}{3}\left(1+4+9+16\right) = 1$

$c \cdot \frac{2}{3} \cdot 30 = 1 \qquad c \cdot 20 = 1 \qquad c = \frac{1}{20} = \underline{\underline{0.05}}$

$p_X(k) = \frac{1}{30} k^2 \qquad k = 1, 2, 3, 4$

b) och c)

k	1	2	3	4
$p_X(k)$	$1/30$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$
$F_X(k)$	$1/30$	$5/30$	$\frac{14}{30}$	1

d) $P(X \leq 2) = p_X(1) + p_X(2) = \frac{1}{30} + \frac{4}{30} = \frac{5}{30} = \underline{\underline{\frac{1}{6}}} \approx 0.166$

e) $E(X) = \sum_{k=1}^{4} k \cdot p_X(k) = 1 \cdot \frac{1}{30} + 2 \cdot \frac{4}{30} + 3 \cdot \frac{9}{30} + 4 \cdot \frac{16}{30}$

$= \frac{1}{30}\left(1+8+27+64\right) = \frac{1}{30} \cdot 100 = \underline{\underline{\frac{10}{3}}} \approx 3,33$

f) $V(X) = E(X^2) - E(X)^2$

$E(X^2) = \sum_{k=1}^{4} k^2 \cdot p_X(k) = 1^2 \cdot \frac{1}{30} + 2^2 \cdot \frac{4}{30} + 3^2 \cdot \frac{9}{30} + 4^2 \cdot \frac{16}{30}$

$= \frac{1}{30}\left(1+16+81+256\right) = \frac{354}{30} = \frac{118}{10} = \underline{\underline{11.8}}$

$V(X) = E(X^2) - E(X)^2 = \frac{118}{10} - \frac{100}{9} = \frac{1062 - 1000}{90} = \frac{62}{90} = \underline{\underline{\frac{31}{45}}} \approx \underline{\underline{0.688}}$

$D(X) = \sqrt{\frac{31}{45}} \approx \underline{0.83}$

g) $Y = 3 \cdot X + 1$

$V(Y) = V(3 \cdot X + 1) = 9 \cdot V(X) = 9 \cdot \frac{31}{45} = \underline{\underline{\frac{31}{5}}} \approx 6.2$

(2.) s.v. S („sent") : Anna kommer för sent tenta 23/10 - 2018

$P(S|A) = 0.01$ $\qquad P(A) = \frac{1}{5}$ slh. att A kommer först

$P(S|B) = 0.03$ $\qquad P(B) = 3/10$ — " — B — " —

$P(S|C) = 0.06$ $\qquad P(C) = 1/2$ — " — C — " —

total sannolikhet:

$$P(A) + P(B) + P(C) = \frac{1}{5} + \frac{3}{10} + \frac{1}{2} = \frac{1}{10}(2+3+5) = \underline{1}$$

a) $P(S) = P(S|A) \cdot P(A) + P(S|B) \cdot P(B) + P(S|C) \cdot P(C)$

$= 0.01 \cdot \frac{1}{5} + 0.03 \cdot \frac{3}{10} + 0.06 \cdot \frac{1}{2}$

$= \frac{1}{500} + \frac{9}{1000} + \frac{6}{200} = \frac{1}{1000}(2 + 9 + 30) = \frac{41}{1000} = \underline{\underline{0.041}}$

Bayes sats

b) $$P(C|S) = \frac{P(S|C) \cdot P(C)}{P(S)} = \frac{0.06 \cdot \frac{1}{2}}{\frac{41}{1000}} = \frac{30}{41} \approx \underline{\underline{73\%}}$$

(3) $X_A \sim N(20,2)$ $X_B \sim N(24,1)$ $X_C \sim N(30,2)$ } sammanlagd tid: $X = X_A + X_B + X_C$

tenta 23/10 - 2018

$X \sim N(74, \sqrt{2^2+1^2+2^2})$

$X \sim N(74, 3)$

$P(X>80) = 1 - P(X<80)$

$= 1 - F_X(80)$

$= 1 - \Phi\left(\frac{80-74}{3}\right) = 1 - \Phi(2) = 1 - 0{,}977 = 0{,}0227$

$P(X>80) = \underline{\underline{2.3\%}}$

(4) s.v. X = antalet felaktiga i en förpackning   tenta 23/10-2018

a) $X \sim Bin(5, 0.2)$

b) $E(X) = n \cdot p = \underline{\underline{1}}$

$V(X) = n \cdot p \cdot (1-p) = 5 \cdot 0.2 \cdot 0.8 = \underline{\underline{0.8}}$

c) $P(X = k) = \binom{n}{k} p^k \cdot (1-p)^{n-k}$

$P(\text{alla fungerar}) = P(X=0) = \binom{5}{0} \cdot 0.2^0 \cdot 0.8^5 = \underline{\underline{0.8^5}} \approx 0.33$

d) $P(X<2) = P(X \le 1) = p_X(0) + p_X(1)$

$= 0.8^5 + \binom{5}{1} \cdot 0.2^1 \cdot 0.8^4 = 0.8^5 + 5 \cdot 0.2 \cdot 0.8^4$

$= 0.8^5 + 0.8^4 = 0.8^4(0.8+1) = \underline{\underline{1.8 \cdot 0.8^4}} \approx 73.7\%$

e) $Y \sim Bin(5, 0.8)$

f) $Y \sim Bin(n, 0.8)$ n obestämd

krav: $P(\text{minst en fungerar}) = P(Y \ge 1) \ge 0.99$

$p_Y(k) = \binom{n}{k} p^k \cdot (1-p)^{n-k}$ $p = 0.8$ n obestämd

$P(Y \ge 1) \ge 0.99$

$1 - P(Y<1) \ge 0.99$

$1 - p_Y(0) \ge 0.99$

$-p_Y(0) \ge -0.01$

$p_Y(0) \le 0.01$ $p_Y(0) = \binom{n}{0} \cdot 0.8^0 \cdot 0.2^n = 0.2^n$

$\boxed{0.2^n \le 0.01}$

(slh. att alla är felaktiga ska vara mindre än 0.01)

$n \cdot \log_{10}(0.2) \le \log_{10}(0.01)$ $\log_{10}(0.2) < 0$!

$n \ge \frac{-2}{\log_{10}(0.2)}$

$n \ge 2.86$

$\underline{\underline{n \ge 3}}$ Kontroll: $0.2^2 = 0.04$

$0.2^3 = 0.008$ o.k.

(5) s.v. X : antalet strömavbrott

$X \sim Po(\mu)$ $\quad \mu = \lambda \cdot t \quad \lambda = \frac{0{,}5}{\text{månad}}$

a) $t = 1$ månad $\quad \mu = 0{,}5$

$$p_X(k) = \frac{\mu^k}{k!} e^{-\mu}$$

$$p_X(0) = \frac{0{,}5^0}{0!} e^{-0{,}5} = \underline{\underline{e^{-\frac{1}{2}}}} = 0{,}606$$

b) $t = 12$ månader $\quad \mu = 6$

$X \sim Po(6)$

$$P(X > 5) = 1 - P(X \leq 5) = 1 - F_X(5)$$

$$= 1 - 0{,}4457 = 0{,}55 = \underline{\underline{55\%}}$$

c) $P(X > 5) = 1 - P(X \leq 5) < 0{,}3$ krav

$\Leftrightarrow \quad P(X \leq 5) \geq 0{,}7$

tabell $\quad \mu \approx 4{,}5$

Kontroll med $\mu = 4{,}5$; dvs $X \sim Po(4{,}5)$

$$P(X > 5) = 1 - P(X \leq 5)$$

$$= 1 - F_X(5) \quad \text{tabell}$$

$$= 1 - 0{,}7029$$

$$= 0{,}297 \leq 30\% \quad \text{o.k.}$$

väntevärdet per månad:

$$\mu_1 = \frac{\mu}{12} = \frac{4{,}5}{12} \approx \underline{\underline{0{,}375}}$$

(6) Skattning av proportion:

$$I_p = \frac{x}{n} \pm \lambda_{\alpha/2} \cdot d \qquad d = \sqrt{\frac{\frac{x}{n}\left(1-\frac{x}{n}\right)}{n}}$$

$\alpha = 0.01$ (99% KI) $\qquad \lambda_{\alpha/2} = \lambda_{0.005} = 2.58$

$n = 200$

$x = 32$

$$d = \sqrt{\frac{\frac{32}{200}\left(\frac{200}{200} - \frac{32}{200}\right)}{200}} = \sqrt{\frac{\frac{32}{200} \cdot \frac{168}{200}}{200}} = \sqrt{\frac{\frac{4}{25} \cdot \frac{21}{25}}{200}}$$

$d = 0.026$

$I_p = 0.16 \pm 2.58 \cdot 0.026$

$I_p = 0.16 \pm 0.067 = (0.09 ; 0.23)$

7. $n = 10$ $\quad X \sim N(\mu, \sigma)$ tenta 23/10 - 2018

$\bar{x} = 0.506$ cm

$s = 0.004$ cm

a) $\alpha = 0.05$

σ okänd, litet stickprov

$I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}$ $\qquad t_{\alpha/2}(n-1) = t_{0.025}(9) = 2.26$

$I_\mu = 0.506 \pm 2.26 \cdot \frac{0.004}{\sqrt{10}}$

$I_\mu = 0.506 \pm 0.0028 = (0.503\, ; 0.509)$

b) Ta större stickprov.

8. *parade* observationer $\quad \sigma$ okänd $\quad$ tenta 23/10 - 2018

$\alpha = 0.05$

$n = 6$

a) $I_{\Delta\mu} = \bar{z} \pm t_{\alpha/2}(n-1) \cdot \frac{s_z}{\sqrt{n}}$

$z = \{6, 7, 9, -4, 12, 10\}$

$\bar{z} = \frac{1}{n}\sum_{i=1}^{6} z_i = \underline{\underline{6.67}} = \frac{20}{3}$

$s_z = \frac{1}{n-1}\sum_{i=1}^{6}(z_i - \bar{z})^2 = \underline{\underline{5.65}}$

$t_{\alpha/2}(n-1) = t_{0.025}(5) = \underline{\underline{2.57}}$

$I_{\Delta\mu} = 6.67 \pm 2.57 \cdot \frac{5.65}{\sqrt{6}}$

$I_{\Delta\mu} = 6.67 \pm 5.93$

$I_{\Delta\mu} = (0.74 ; 12.6)$

b) Ja, det finns en signifikant skillnad. Konfidensintervallet för $\Delta\mu$ innehåller inte noll.

(9.) $I_{\Delta\mu}$ sökes σ_x, σ_y okända men lika stora $\sigma_x = \sigma_y = \sigma$
små stickprov.

a) $I_{\Delta\mu} = \bar{x} - \bar{y} \pm t_{\alpha/2}(f) \cdot d$ $f = n_x + n_y - 2$

$$d = s_p \cdot \sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \qquad s_p = \sqrt{\frac{(n_x-1)\cdot s_x^2 + (n_y-1)\cdot s_y^2}{f}}$$

$$s_p = \sqrt{\frac{6 \cdot 17.5^2 + 4 \cdot 15.2^2}{10}}$$

$f = 10$

$t_{\alpha/2}(f) = t_{0.025}(10) = 2.23$

$s_p = 16.65$

$$d = 16.65 \cdot \sqrt{\frac{1}{7} + \frac{1}{5}} = 16.65 \cdot 0.585 = 9.75$$

$I_{\Delta\mu} = 290 - 221 \pm 2.23 \cdot 9.75$

$I_{\Delta\mu} = 69 \pm 21.74$

$I_{\Delta\mu} = (47.2 \,;\, 90.7)$

b) Ja, konfidensintervallet innehåller inte noll.
Grupp x (stål) har större tänjbar styrka.