Probabilities, PDF, and CDF visualised

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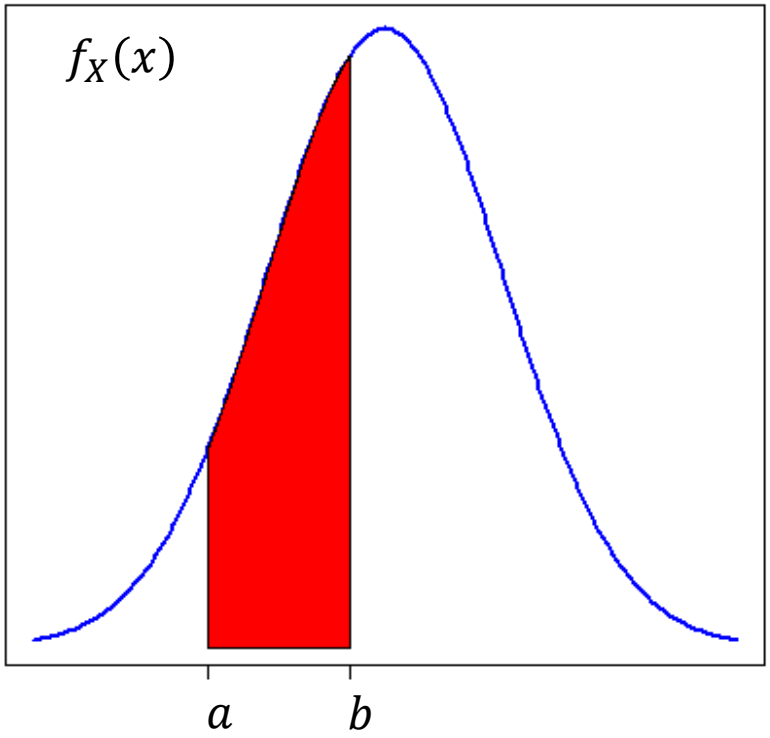
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The probability that the outcome of a continuous random variable X ends up between a and b equals the area under the density function (PDF) between a and b (red in the graph below). This is also equal to $F_X(b) - F_X(a)$ because $F_X(b)$ is the area left of b, and $F_X(a)$ is the area left of a.

$$P(a < X \leq b) = F_X(b) - F_X(a) = \int_a^b f(x)\,dx$$

$F_X(x) = P(X \leq x)$ **C**umulative **D**istribution **F**unction (CDF)

$f_X(x)$ **P**robability **D**ensity **F**unction (PDF)

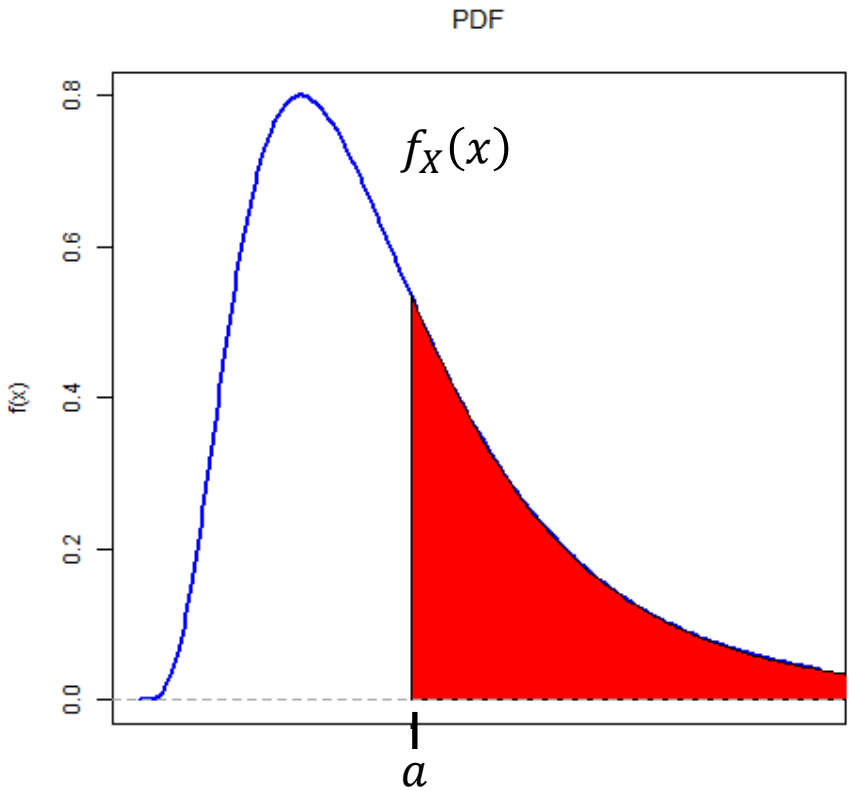


The probability that the outcome of a continuous random variable X ends up at values bigger than a equals the area under the density function (PDF) to the right of a (red area in the graph). This is also equal to $1 - F_X(a)$ because $F_X(a)$ is the area left of a, and the total area under the curve is 1.

$$P(X > a) = 1 - F_X(a) = \int_a^{\infty} f(x)\,dx$$

$F_X(a) = P(X \leq a)$ **C**umulative **D**istribution **F**unction (CDF)

$f_X(x)$ **P**robability **D**ensity **F**unction (PDF)



The probability that the outcome of a continuous random variable X ends up at values lower than a equals the area under the density function (PDF) to the left of a (red area in the graph). This is also equal to $F_X(a)$ because $F_X(a)$ is just the area left of a.

$$P(X \leq a) = F_X(a) = \int_{-\infty}^{a} f(x)\,dx$$

$F_X(a) = P(X \leq a)$ **C**umulative **D**istribution **F**unction (CDF)

$f_X(x)$ **P**robability **D**ensity **F**unction (PDF)

