Tenta 2009-08-20

1.

k	1	2	3	4
$p_X(k)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

a)

k	1	2	3	4
$F_X(k)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10}$	1

b) $E(X) = \sum k \cdot p_X(k) = \dfrac{1+4+9+16}{10} = \underline{\underline{3}}$

c) $E(X^2) = \dfrac{1+8+27+64}{10} = 10$

$V(X) = E(X^2) - \mu^2 = 10 - 3^2 = \underline{\underline{1}}$

d) $E(Y) = E(2X^2) = \sum 2k^2 \cdot p_X(k)$

$= 2\,E(X^2) = 2 \cdot 10 = 20$

e)

k	$(=2\cdot 1^2)$ 2	$(=2\cdot 2^2)$ 8	$(=2\cdot 3^2)$ 18	$(=2\cdot 4^2)$ 32
$p_Y(k)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

$P(Y > 10) = \dfrac{7}{10} = \underline{\underline{70\%}}$

2

2

S: sjukdom $\quad P(S) = 0.1 \quad \Rightarrow P(S^c) = 0.9$

N: negativ $\quad P(N|S) = 0.2 \quad$ för högt

P: positiv $\quad P(P|S^c) = 0.1 \quad$ — " —

$P(P|S) = 0.8$

Söker: $P(S|P)$?

sjuk $\quad$ positiv

$$\boxed{P(S|P) = \frac{P(P|S)\cdot P(S)}{P(P)}}$$

Bayes

total sll.

N.R.

$$P(P) = P(S)\cdot P(P|S) + P(S^c)\cdot P(P|S^c)$$
$$= 0.1 \cdot 0.8 + 0.9 \cdot 0.1 = 0.08 + 0.09 = \underline{\underline{0.17}}$$

$$P(S|P) = \frac{P(P|S)\cdot P(S)}{P(P)}$$

$$= \frac{0.8 \cdot 0.1}{0.17} = \frac{0.08}{0.17} = \frac{8}{17} = \underline{\underline{0.47}}$$

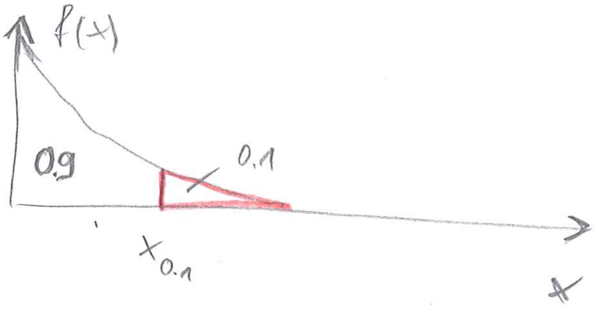
3.

$f(x) = e^{-x}, \quad x > 0$

$X \sim Exp(1)$

$$\boxed{F_X(x) = 1 - e^{-x}}$$

$= \int_0^x f(t)\,dt = -e^{-t}\Big|_0^x$



$F_X(x_{0.1}) = 0.9$ (find)

$1 - e^{-x_{0.1}} = 0.9$

$e^{-x_{0.1}} = 0.1$

$-x_{0.1} = \ln 0.1$

$x_{0.1} = \underline{\underline{2.303}}$

4. R: ren $\quad P(R) = 0.1$

a) $X \sim Bin(n, p)$ $\quad X$: antalet rena kristaller

$X \sim Bin(n, 0.1)$

b) $X \sim Bin(10, 0.1)$ $\quad n = 10$

$$P(X=1) = p_X(1) = \binom{10}{1} 0.1^1 \, 0.9^9 = 0.387 = \underline{\underline{38.7\%}}$$

c) $X \sim Bin(10, 0.1)$

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [p_X(0) + p_X(1)]$$

$$= 1 - [0.9^{10} + 0.9^9] = \underline{\underline{26,39\%}}$$

d) $\underbrace{P(X \geq 1)}_{\text{sek. att få minst en}} > 0.9$ $\quad$ bestäm n!

$$1 - P(X < 1) > 0.9$$

$$1 - p_X(0) > 0.9$$

$$p_X(0) < 0.1$$

$$\binom{n}{0} p^0 \underbrace{(1-p)}^{n} < 0.1$$

$$0.9^n < 0.1$$

$$n \ln 0.9 < \ln 0.1 \qquad \ln 0.9 < 0$$

$$n > \frac{\ln 0.1}{\ln 0.9} = 21.8$$

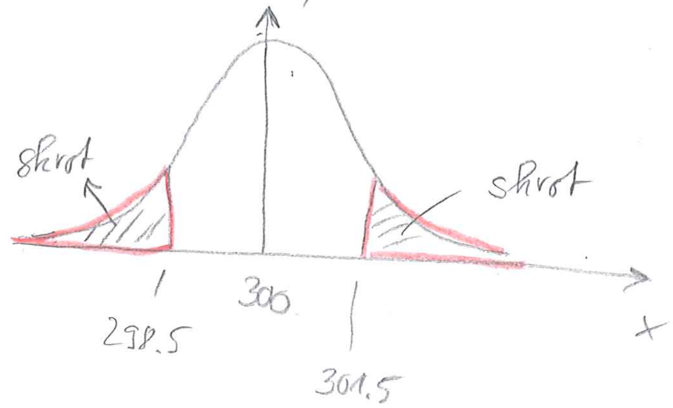
$$\underline{n \geq 22}$$

5. X: diameter

$X \sim N(\mu, \sigma^2)$

$X \sim N(300, 1)$

a)



$P(|X-\mu| > 1.5)$ söles

$= P(X < 298.5) + P(X > 301.5)$ röd

eller

$= 1 - P(298.5 < X \leq 301.5)$

$= 1 - [F_X(301.5) - F_X(298.5)]$

$= 1 - F_X(301.5) + F_X(298.5)$

$= 1 - \phi\left(\frac{301.5 - 300}{1}\right) + \phi\left(\frac{298.5 - 300}{1}\right)$

$= 1 - \phi(1.5) + \phi(-1.5)$

$= 1 - \phi(1.5) + 1 - \phi(1.5)$

$= 2 - 2 \cdot \phi(1.5)$

$= 2 - 2 \cdot 0.9332$

$= \underline{0.1336} \approx 13.4\%$ skrot

5. fortsättning

$$\underbrace{P\left(|X-\mu| > 1.5\right)}_{\text{„ skrot"}} \leq 0.05$$ krävs (σ sökes)

gäller $\sigma = 1$

$\Downarrow$

$$1 - P(298.5 < X \leq 301.5) \leq 0.05$$

samma räkning, men σ inte 1

$$1 - \phi\left(\frac{1.5}{\sigma}\right) + \phi\left(\frac{-1.5}{\sigma}\right) \leq 0.05$$

$$1 - \phi\left(\frac{1.5}{\sigma}\right) + 1 - \phi\left(\frac{1.5}{\sigma}\right) \leq 0.05$$

$$2 - 2 \cdot \phi\left(\frac{1.5}{\sigma}\right) \leq 0.05$$

$$-2 \cdot \phi\left(\frac{1.5}{\sigma}\right) \leq -1.95$$

$$\phi\left(\frac{1.5}{\sigma}\right) \geq 0.975$$

$\phi(x)$ växer monotont med x

$$\frac{1.5}{\sigma} \geq 1.96$$ tabell

$$\sigma \leq \frac{1.5}{1.96}$$

$$\sigma \leq \underline{\underline{0.765}}$$

för att få mindre än 5% skrot

6.

$n = 14$

$\bar{x} = 32{,}132$

$s = 2{,}595$

$\alpha = 0.01$

$$\boxed{I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}}$$

$$= 32{,}132 \pm t_{0.005}(13) \cdot \frac{2{,}595}{\sqrt{14}}$$

$t_{0.005}(13) = 3.0123$

$$= 32132 \pm 2089.16$$

$$= \underline{(30042,\ 34221)}$$

$n_x = 100$

$n_y = 100$

$\bar{x} = 798 \qquad s_x = 2.8$

$\bar{y} = 826 \qquad s_y = 3.0$

$$\boxed{I_{\Delta\mu} = \bar{x} - \bar{y} \pm \lambda_{\alpha/2} \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}}$$

stora stickprov, normalapproximation

$= 28 \pm 0.8$

$= \underline{(27.2 \ , \ 28.8)}$

jmf: x har längre livstid (med 95% konfidens) parade stickprov

modell

$$\boxed{I_\Delta = \bar{z} \pm t_{\alpha/2}(n-1) \cdot \frac{s_z}{\sqrt{n}}}$$

$\bar{z} = 2.57$

$s_z = 2.37$

$t_{0.025}(6) = 2.45$

$$I_\Delta = 2.57 \pm 2.45 \cdot \frac{2.37}{\sqrt{7}}$$

$$= 2.57 \pm 2.195$$

$$= \underline{(0.375,\ 4.765)}$$

skillnad finns!