Tenta 22/8 2018

1.

	k	1	2	3	4	5
a)	$p_X(k)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
b)	$F_X(k)$	$\frac{1}{15}$	$\frac{3}{15}$	$\frac{6}{15}$	$\frac{10}{15}$	1

c) $P(X<3) = p_X(1) + p_X(2) = \frac{1}{15} + \frac{2}{15} = \underline{\underline{\frac{1}{5}}}$

$P(X \geq 2) = 1 - p_X(1) = \frac{15}{15} - \frac{1}{15} = \underline{\underline{\frac{14}{15}}}$

d) $E(X) = \sum_k k \cdot p_X(k) = 1 \cdot \frac{1}{15} + 2 \cdot \frac{2}{15} + 3 \cdot \frac{3}{15} + 4 \cdot \frac{4}{15}$

$+ 5 \cdot \frac{5}{15} = \frac{55}{15} = \underline{\underline{\frac{11}{3}}}$

e) $V(X) = E(X^2) - \mu^2 \qquad \mu^2 = \left(\frac{11}{3}\right)^2 = \frac{121}{9}$

$E(X^2) = 1^2 \cdot \frac{1}{15} + 2^2 \cdot \frac{2}{15} + 3^2 \cdot \frac{3}{15} + 4^2 \cdot \frac{4}{15} + 5^2 \cdot \frac{5}{15}$

$= \frac{1}{15} + \frac{8}{15} + \frac{27}{15} + \frac{64}{15} + \frac{125}{15} = \frac{225}{15} = \underline{\underline{15}}$

$V(X) = 15 - \frac{121}{9} = \frac{135}{9} - \frac{121}{9} = \underline{\underline{\frac{14}{9}}}$

$D(X) = \sqrt{\frac{14}{9}}$

f) $E(Y) = E(2 + 3 \cdot X) = 2 + 3 \cdot E(X) = 2 + 3 \cdot \frac{11}{3} = \underline{\underline{13}}$

linjär transformation

(2) $K =$ „klarar"

u_1 : uppgift ett

u_a : annan uppgift

$$P(u_1) = \frac{1}{4} \qquad P(K|u_1) = \frac{6}{10}$$

$$P(u_a) = \frac{3}{4} \qquad P(K|u_a) = \frac{4}{10}$$

a) $$P(K) = P(K|u_1) \cdot P(u_1) + P(K|u_a) \cdot P(u_a)$$
$$= \frac{6}{10} \cdot \frac{1}{4} + \frac{4}{10} \cdot \frac{3}{4} = \frac{9}{20} = \underline{\underline{0.45}}$$

b) givit: händelse K („klarat")

$$P(u_1|K) = \frac{P(K|u_1) \cdot P(u_1)}{P(K)} = \frac{\frac{6}{10} \cdot \frac{1}{4}}{\frac{9}{20}} = \underline{\underline{\frac{1}{3}}}$$

(Bayes sats)

$P(u_1|K)$ sökes = sell. att u_1 dragits givit att hen klarade tentan

③ $p = \frac{1}{4} \quad n = 12$

a) $X \sim Bin(n, p)$

$X \sim Bin(12, \frac{1}{4})$

Wh.-funktion

$p_X(k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$

b) $k = 12$

$p_X(12) = \binom{12}{12}\left(\frac{1}{4}\right)^{12}\left(\frac{3}{4}\right)^0 = \left(\frac{1}{4}\right)^{12}$

c) $P(X < 2) = p_X(0) + p_X(1)$

$= \binom{12}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{12} + \binom{12}{1}\left(\frac{1}{4}\right)^1 \cdot \left(\frac{3}{4}\right)^{11}$

$= \left(\frac{3}{4}\right)^{12} + 12 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^{11} = \left(\frac{3}{4}\right)^{12} + 3 \cdot \left(\frac{3}{4}\right)^{11} = \underline{\underline{0.1584}}$

d) $p_X(0) = \binom{12}{0} \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{12} = \left(\frac{3}{4}\right)^{12} \approx \underline{\underline{0.0317}}$

e) $E(X) = n \cdot p = 12 \cdot \frac{1}{4} = \underline{\underline{3}}$

$V(X) = n \cdot p \cdot (1-p) = 12 \cdot \frac{1}{4} \cdot \frac{3}{4} = \underline{\underline{\frac{9}{4}}}$

(4.) $X \sim Po(\mu)$ $E(X) = \mu = 3$

$$p_X(k) = \frac{\mu^k}{k!} e^{-\mu}$$

a) $p_X(0) = \frac{3^0}{0!} e^{-3} = \underline{\underline{e^{-3}}}$

b) $\mu = 9$! (för 3 sekunder $\mu = \lambda \cdot t = \frac{3}{s} \cdot 3s = 9$)

$p_X(0) = \frac{9^0}{0!} e^{-9} = \underline{\underline{e^{-9}}}$

c) $\mu = 3$ (en sekund)

$P(X > 6) = 1 - P(X \leq 6) = 1 - F_X(6)$ tabell

$= 1 - 0.9665 = \underline{\underline{0.0335}}$

(5.) $X \sim Exp(\lambda) \qquad E(X) = 3 = \frac{1}{\lambda} \qquad \lambda = \frac{1}{3}$

$$f_X(x) = \lambda \cdot e^{-\lambda \cdot x} \qquad x > 0$$

$$F_X(x) = 1 - e^{-\lambda \cdot x} \qquad x > 0$$

a) $P(X > 9) = 1 - P(X \leq 9) = 1 - F_X(9)$

$$= 1 - \left[1 - e^{-\frac{1}{3}9}\right] = e^{-3} = \underline{\underline{0.0498}}$$

b) $P(3 < X \leq 6) = F_X(6) - F_X(3)$

$$= 1 - e^{-\frac{1}{3} \cdot 6} - \left[1 - e^{-\frac{1}{3} \cdot 3}\right]$$

$$= e^{-1} - e^{-2} \approx \underline{\underline{0.2325}}$$

c) $P(X > 9) = 1 - P(X \leq 9) = 1 - F_X(9) = e^{-\lambda \cdot 9}$

$P(X > 9) \leq 0.001$ bestäm λ, sedan $E(X)$

$$e^{-\lambda \cdot 9} \leq 0.001 \quad | \ln(\cdot)$$

$$-\lambda \cdot 9 \leq \ln(10^{-3})$$

$$-\lambda \cdot 9 \leq -3 \cdot \ln(10) \quad | \cdot (-1)$$

$$\lambda \cdot 9 \geq 3 \cdot \ln(10)$$

$$\lambda \geq \frac{1}{3} \ln(10)$$

$$\frac{1}{\lambda} \leq \frac{3}{\ln(10)} \qquad E(X) = \frac{1}{\lambda} \text{ används}$$

$$E(X) \leq \frac{3}{\ln(10)} \approx 1.3$$

6. $X \sim N(\mu, \sigma) \quad X \sim N(3, 0.15)$

a) $P(X > 3.15) = 1 - P(X \le 3.15) = 1 - F_X(3.15)$
$= 1 - \phi\left(\frac{3.15-3}{0.15}\right) = 1 - \phi(1) = 1 - 0.8413 \approx \underline{\underline{16\%}}$

b) $P(2.7 < X \le 3.3) = F_X(3.3) - F_X(2.7) = \phi\left(\frac{3.3-3}{0.15}\right) - \phi\left(\frac{2.7-3}{0.15}\right)$
$= \phi(2) - \phi(-2) = \phi(2) - [1 - \phi(2)]$
$= 2 \cdot \phi(2) - 1 = 2 \cdot 0.9772 - 1 = 0.9544 \approx \underline{\underline{95\%}}$

(är lika med $P(\mu - 2\sigma < X \le \mu + 2\sigma)$

c) $\bar{X}_n \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \quad \bar{X}_{25} \sim N\left(3, \frac{0.15}{\sqrt{25}}\right) \quad \bar{X}_{25} \sim N(3, 0.03)$

$P(\bar{X}_{25} > 3.15) = 1 - F_{\bar{X}_{25}}(3.15) = 1 - \phi\left(\frac{3.15-3}{0.03}\right)$
$= 1 - \underbrace{\phi(5)}_{\approx 1} \approx \underline{\underline{0}}$

d) $X_i \sim N(\mu, \sigma) \quad \sum_{i=1}^{n} X_i \sim N(n \cdot \mu, \sqrt{n} \cdot \sigma) \quad n = 100$

$X_{100} = \sum_{i=1}^{100} X_i \sim N(300, 1.5)$

$P(X_{100} > 303) = 1 - P(X_{100} \le 303) = 1 - F_{X_{100}}(303)$
$= 1 - \phi\left(\frac{303-300}{1.5}\right) = 1 - \phi(2)$
$= 1 - 0.9772 = 0.0228 \approx \underline{\underline{2.3\%}}$

7.

$I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}$ σ okänd, litet stickprov

$n = 9$

$\alpha = 0.05$ (95% KI)

$t_{0.025}(8) = 2.306$

$\bar{x} = 11.11$

$s^2 = 0.67$

$I_\mu = 11.11 \pm 2.31 \cdot \frac{\sqrt{0.67}}{3}$

$= 11.11 \pm 0.6285$

$= 11.11 \pm 0.63$

$I_\mu = (10.47 ; 11.74)$

(8.) $\alpha = 0.05$ parade stickprov

$z_i = \{4, 5, 9, -3, 1, 12\}$

$n = 6$

a) $I_{\Delta\mu} = \bar{z} \pm t_{\alpha/2}(n-1) \cdot \frac{s_z}{\sqrt{n}}$

$\bar{z} = 4.67$

$s_z^2 = 29.07$

$s_z = 5.39$ $\qquad t_{0.025}(5) = 2.57$

$I_{\Delta\mu} = 4.67 \pm 2.57 \cdot \frac{5.39}{\sqrt{6}}$

$= 4.67 \pm 5.65$

b) Nej. (KI innehåller nollan)