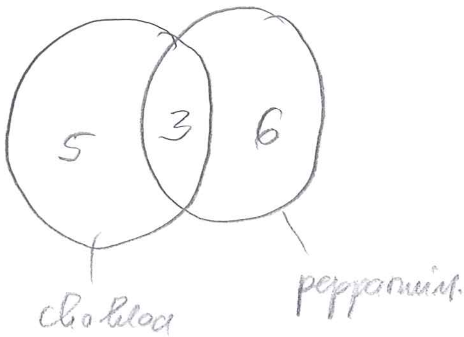
Tenta 2009-01-14

1

a)



totalt = 30

b) $P = \frac{11}{30}$

2 K_1, K_2, K_3, K_4 C_1, C_2

· ohne Wahl
· Reihenfolge ega

$n = 2$

a) K_1K_2, K_1K_3, K_1K_4, K_1C_1, K_1C_2

K_2K_3, K_2K_4, K_2C_1, K_2C_2

K_3K_4, K_3C_1, K_3C_2

K_4C_1, K_4C_2

C_1C_2

$m = 15$

$\binom{6}{2} = \frac{6!}{4!\,2!} = \frac{6 \cdot 5}{2} = \underline{\underline{15}}$

b) $P(KK) = \frac{6}{15}$

c) $P(K*) = \frac{14}{15}$

d) $P(CC) = \frac{1}{15}$

e) $P(KC) = \frac{8}{15}$

3

K – karamell $\quad P(K) = 0.7$

T – tuggummi $\quad P(T) = 0.3 \qquad T = K^c$

C – citronsmak

$P(C|K) = 0.1$

$P(C|T) = 0.2$

a) $P(K) = 0.7$

$$P(C) = P(K) \cdot P(C|K) + P(T) \cdot P(C|T)$$
$$= 0.7 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.06 = \underline{\underline{0.13}}$$

b) $$P(T|C) = \frac{P(C|T) \cdot P(T)}{P(C)} = \frac{0.2 \cdot 0.3}{0.13} = \frac{0.06}{0.13} = \frac{6}{13}$$

$$P(K|C) = \frac{P(C|K) \cdot P(K)}{P(C)} = \frac{0.1 \cdot 0.7}{0.13} = \frac{0.07}{0.13} = \frac{7}{13}$$

c) C och K

$P(C|K) \overset{?}{=} P(C)$

$0.1 \neq 0.13$ ej oberoende

eller $P(K|C) \overset{?}{=} P(K)$

$\frac{7}{13} \neq 0.7$ ej oberoende

eller $$P(C|K) = \frac{P(C \cap K)}{P(K)}$$

$$P(C \cap K) = P(K) \cdot P(C|K) = 0.7 \cdot 0.1 = \underline{\underline{0.07}}$$

karamell med citronsmak

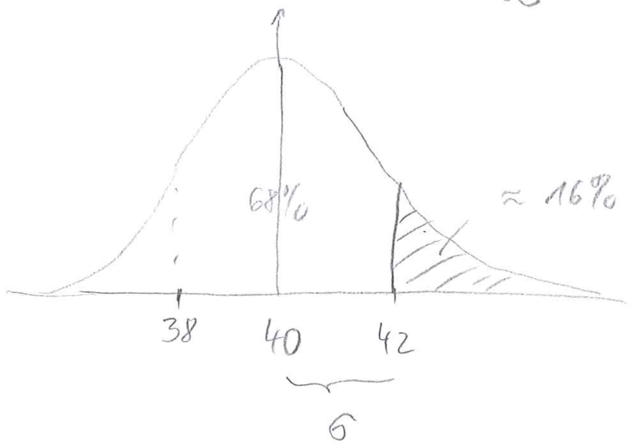
$P(C \cap K) \overset{?}{=} P(C) \cdot P(K)$

$0.07 \neq 0.13 \cdot 0.7$ ej oberoende

(4) (3)

$X \sim N(40, 2^2)$ $\mu = 40$, $\sigma = 2$

a)



b) $P(X > 42) = 1 - P(X \leq 42) = 1 - F_X(42)$

$= 1 - \Phi\left(\frac{42-40}{2}\right) = 1 - \Phi(1) = 1 - 0.8413$

$\cong \underline{\underline{16\%}}$

c) $\mu \pm 2\sigma \rightarrow \approx 95\%$

$P(36 < X \leq 44) = F_X(44) - F_X(36) = \Phi\left(\frac{44-40}{2}\right) - \Phi\left(\frac{36-40}{2}\right)$

$= \Phi(2) - \Phi(-2) = \Phi(2) - [1 - \Phi(2)] = 2\Phi(2) - 1$

$= 2 \cdot 0.9772 - 1 = 0.9544 \approx \underline{\underline{95\%}}$

d) $P(X < 36 \cup X > 44) = 1 - P(36 < X \leq 44) \rightarrow$ c)

$= 1 - 0.9544 \approx \underline{\underline{5\%}}$

e) $\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right) \Rightarrow \bar{X}_{100} \sim N(40, (0.2)^2)$ $\quad (0.2)^2 = 0.04 = \frac{2^2}{100}$

$P(\bar{X}_{100} > 40.4) = 1 - P(\bar{X}_{100} \leq 40.4) = 1 - F_{\bar{X}_{100}}(40.4)$

$= 1 - \Phi\left(\frac{40.4-40}{0.2}\right) = 1 - \Phi(2) = 1 - 0.9772$

$= 0.0228 \approx 2.3\%$

5

a) $X \sim Bin(n, p)$ $\quad X$: antalet utgångna

b) $\left.\begin{matrix} n = 100 \\ x = 20 \end{matrix}\right\}$ $\quad p^* = \frac{20}{100} = 0.2$

c) Normalappr. (n stor)

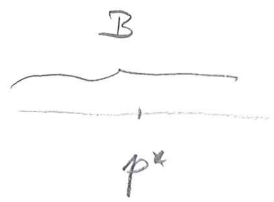
$$I_p = p^* \pm \lambda_{\alpha/2} \sqrt{\frac{p^*(1-p^*)}{n}}$$

$$I_p = 0.2 \pm 1.96 \sqrt{\frac{0.2 \cdot 0.8}{100}}$$

$$= 0.2 \pm 0.0784 = (0.1216, 0.2784)$$

$$= (12\%, 28\%)$$

d) bredd: B



$$B = 2 \cdot \lambda_{\alpha/2} \sqrt{\frac{p^*(1-p^*)}{n}} = 2 \cdot 0.0784 = 0.1568$$

denna bredd har vi

bestäm n!

$$0.1568 = 2 \cdot \overbrace{\underset{= 2.576}{\lambda_{0.005}}}^{99\% \text{ KI}} \cdot \sqrt{\frac{0.2 \cdot 0.8}{n}}$$

$$\frac{0.1568}{2 \cdot 2.576} = \sqrt{\frac{0.2 \cdot 0.8}{n}}$$

$$9.263 \cdot 10^{-4} = \frac{0.2 \cdot 0.8}{n} \qquad n = 173$$

6

$X \sim N(\mu, \sigma^2)$ σ okänd

$n = 6$

$$\boxed{I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}}$$

$\bar{x} = 41.13$ $t_{0.025}(5) = 2.571$

$s = 2.073$

$$I_\mu = 41.13 \pm 2.571 \cdot \frac{2.073}{\sqrt{6}}$$

$$= 41.13 \pm 2.176$$

$$= \underline{(38.95, 43.31)}$$ 95% KI för sockerhalten

7.

parade observationer

modellantaganden

$x_i \sim N(\mu_i, \sigma_x^2)$
$y_i \sim N(\mu_i + \Delta, \sigma_y^2)$

- båda N
- systematiskt forskjuter Δ
- σ_x, σ_y

$$\boxed{I_\Delta = \bar{z} \pm t_{\alpha/2}(n-1) \cdot \frac{s_z}{\sqrt{n}}}$$

$\bar{z} = 1.88$ $\quad t_{0.025}(4) = 2.776$

$s_z = 2$

$$I_\Delta = 1.88 \pm 2.776 \cdot \frac{2}{\sqrt{5}}$$

$$= 1.88 \pm 2.48$$

$$= \underline{(-0.6,\ 4.36)}$$

n måste förlängas!

⑦

a) $\boxed{\beta^* = \frac{S_{xy}}{S_{xx}}}$ $x = (8, 12, 18, 27, 32)$

$y = (7, 12, 21, 26, 37)$

lutning $n = 5$

$$S_{xy} = \sum x_i y_i - n\bar{x}\bar{y} = 2334 - 5 \cdot 18.4 \cdot 20.6 = 438.8$$

$$S_{xx} = \sum x_i^2 - n\bar{x}^2 = 2040 - 5 \cdot 18.4^2 = 347.2$$

$$S_{yy} = \sum y_i^2 - n\bar{y}^2 = 2679 - 5 \cdot 20.6^2 = 557.2$$

$\beta^* = 1.264$ ganska långt bort från 1

b) formelsamlingen

$$\boxed{I_\beta = \beta^* \pm t_{\alpha/2}(n-2) \cdot \frac{s_r}{\sqrt{S_{xx}}}}$$

σ okänd : t
men $t(n-2)$
σ skattas med s_r

$$\boxed{s_r^2 = \frac{Q_0}{n-2}} \quad \boxed{Q_0 = S_{yy} - \frac{S_{xy}^2}{S_{xx}}}$$

$$Q_0 = 557.2 - \frac{438.8^2}{347.2} = 2.634$$

$s_r = 0.937$

$$I_\beta = 1.264 \pm \underbrace{t_{0.025}(3)}_{=3.1824} \cdot \frac{0.937}{\sqrt{347.2}} = 1.264 \pm 0.16$$

$I_\beta = (1.1, 1.42)$ innehåller inte 1 dåligt.