Tenta MS0065 HT17

(1.)

k	1	2	3	4
a) $p_X(k)$	0.3	0.2	0.4	0.1
b) $F_X(k)$	0.3	0.5	0.9	1

c) $P(X<3) = p_X(1) + p_X(2) = \underline{\underline{0.5}}$

$P(X \geq 2) = p_X(2) + p_X(3) + p_X(4) = \underline{\underline{0.7}}$

d) $E(X) = 1 \cdot 0.3 + 2 \cdot 0.2 + 3 \cdot 0.4 + 4 \cdot 0.1 = \underline{\underline{2.3}}$

e) $V(X) = E(X^2) - \mu^2$

$E(X^2) = 1^2 \cdot 0.3 + 2^2 \cdot 0.2 + 3^2 \cdot 0.4 + 4^2 \cdot 0.1 = 6.3$

$V(X) = 6.3 - 2.3^2 = 1.01$

$D(X) = \underline{\underline{1.005}}$

deluppgift e)

f) $E(Y) = E(2 \cdot X^2) = 2 \cdot E(X^2) = 2 \cdot 6.3 = \underline{\underline{12.6}}$

Tenta MS0065 HT17

där $\mu = \lambda \cdot t$

(2) $X \sim Po(\mu)$ eller $X \sim Po(\lambda \cdot t)$ $\lambda = \frac{2}{s}$ (per sekund)

a) $t = 1s$ $\lambda \cdot t = \frac{2}{s} \cdot 1s = 2$ $X \sim Po(2)$ dvs. $\mu = 2$

$$p_X(k) = \frac{\mu^k}{k!} e^{-\mu}$$

$$p_X(0) = \frac{2^0}{0!} e^{-2} = e^{-2} = \underline{\underline{0.135}}$$

b) $t = 3s$ $\mu = \lambda \cdot t = \frac{2}{s} \cdot 3s = 6$ $X \sim Po(6)$

$$p_X(0) = \frac{6^0}{0!} e^{-6} = \underline{\underline{0.0025}}$$

c) $t = 1s$ $\mu = \lambda \cdot t = 2$ $X \sim Po(2)$

$$P(X > 6) = 1 - P(X \leq 6)$$

$$= 1 - F_X(6) \quad \text{tabell}$$

$$= 1 - 0.9955$$

$$= \underline{\underline{0.0045}}$$

Tenta MS0065 HT17

3. $X \sim Exp(\lambda)$

$E(X) = \frac{1}{\lambda} = 4 \quad \rightsquigarrow \lambda = \frac{1}{4}$

$F_X(x) = 1 - e^{-\lambda \cdot x}$

a) $P(X > 10) = 1 - P(X \leq 10)$

$= 1 - F_X(10)$

$= 1 - \left[1 - e^{-\frac{1}{4} \cdot 10}\right] = e^{-2.5} = \underline{\underline{0.082}}$

b) $E(X) = \frac{1}{\lambda} = 2 \quad \rightsquigarrow \lambda = \frac{1}{2}$

$P(X > 10) = 1 - F_X(10)$

$= 1 - \left[1 - e^{-\frac{1}{2} \cdot 10}\right] = e^{-5} = \underline{\underline{0.0067}}$

c) $P(X > 10) = 0.001$

$1 - F_X(10) = 0.001$

$e^{-\lambda \cdot 10} = 0.001$

$-\lambda \cdot 10 = \ln(0.001)$

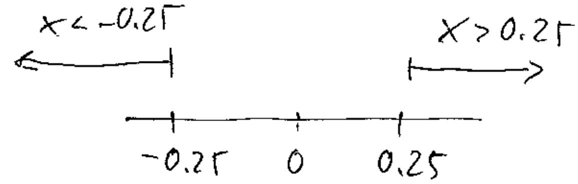
$\lambda = \frac{-\ln(0.001)}{10} = 0.69$

$E(X) = \frac{1}{\lambda} = \frac{1}{0.69} \approx \underline{\underline{1.45}}$

Tenta MS0065 HT17

(4.) $X \sim N(\mu, \sigma)$

$X \sim N(0, 0.5)$



a) $P(|X| > 0.25) = 1 - P(-0.25 < X \leq 0.25)$

$= 1 - \left[F_X(0.25) - F_X(-0.25)\right]$

$= 1 - \left[\Phi\left(\frac{0.25-0}{0.5}\right) - \Phi\left(\frac{-0.25-0}{0.5}\right)\right]$

$= 1 - \Phi\left(\frac{1}{2}\right) + \Phi\left(-\frac{1}{2}\right)$

$= 1 - \Phi\left(\frac{1}{2}\right) + 1 - \Phi\left(\frac{1}{2}\right)$

$= 2 - 2 \cdot \Phi\left(\frac{1}{2}\right) = 2 - 2 \cdot 0.6915 = \underline{\underline{0.617}}$

några beräknade: $P(X > 0.25) = 1 - \Phi\left(\frac{1}{2}\right) = \underline{0.3085}$ (gäller)

b) $\bar{X}_n \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ $n = 25$ $\bar{X}_n \sim N(0, 0.1)$

$P(|\bar{X}_n| > 0.25) = 1 - P(|X_n| \leq 0.25)$

$= 1 - P(-0.25 < X_n \leq 0.25)$

$= 1 - \left[F_{\bar{X}_n}(0.25) - F_{\bar{X}_n}(-0.25)\right]$

$= 1 - \left[\Phi\left(\frac{0.25-0}{0.1}\right) - \Phi\left(\frac{-0.25-0}{0.1}\right)\right]$

$= 1 - \Phi(2.5) - \Phi(-2.5)$

$= 2 - 2 \cdot \Phi(2.5) = 2 - 2 \cdot 0.9938 = \underline{\underline{0.0124}}$

några beräknade $P(\bar{X} > 0.25) = 1 - \Phi(2.5) = \underline{0.0062}$ (gäller)

Tenta MS0065 HT17

(4.)

c) $P(|\bar{X}_n| > 0.25) = 0.003$

$1 - P(|\bar{X}_n| \leq 0.25) = 0.003$

$1 - P(-0.25 < \bar{X}_n \leq 0.25) = 0.003$

$1 - \left[F_{\bar{X}_n}(0.25) - F_{\bar{X}_n}(-0.25) \right] = 0.003$

$1 - \left[\phi\left(\frac{0.25 - 0}{0.5/\sqrt{n}} \right) - \phi\left(\frac{-0.25 - 0}{0.5/\sqrt{n}} \right) \right] = 0.003$

$2 - 2 \cdot \phi\left(\frac{\sqrt{n}}{2} \right) = 0.003$

$1 - \phi\left(\frac{\sqrt{n}}{2} \right) = 0.0015$

$\phi\left(\frac{\sqrt{n}}{2} \right) = 0.9985$ tabell

$\frac{\sqrt{n}}{2} = 2.96$

$n = \underline{\underline{35}}$

några beräknader

$P(\bar{X}_n > 0.25)$

$\rightarrow \quad \phi\left(\frac{\sqrt{n}}{2} \right) = 0.997$

$\frac{\sqrt{n}}{2} = 2.75$

$n = \underline{31}$ gäller!

Tenta MS0065 HT17

(5.) $X \sim N(\mu, \sigma)$ σ okänd litet stickprov

$$I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}$$

$\alpha = 0.05$

$n = 9$ $t_{0.025}(8) = 2.3060$

$$I_\mu = 11.1 \pm 2.306 \cdot \frac{\sqrt{0.039}}{3}$$

$$= 11.1 \pm 0.153 = (10.93; 11.23)$$

(6.) $X \sim N$ parade stickprov

a) $$I_{\Delta\mu} = \bar{z} \pm t_{\alpha/2}(n-1) \cdot \frac{s_z}{\sqrt{n}}$$ $z_i = y_i - x_i$

$z = (0.1; 0.15; 0.05; -0.1; 0.25)$

$\bar{z} = 0.09$ $s_z^2 = 0.01675$ $s_z = 0.1294$

$\alpha = 0.05$ $t_{0.025}(4) = 2.7764$

$n = 5$

$$I_{\Delta\mu} = 0.09 \pm 2.7764 \cdot \frac{0.1294}{\sqrt{5}}$$

$$= 0.09 \pm 0.161 = (-0.071; 0.251)$$

b) nej intervallet innehåller 0.

Tenta MS0065 HT17

(7) a) $X \sim Bin(n, p)$ $X \sim Bin(100, p)$

$Y \sim Bin(n, p)$ $Y \sim Bin(100, p)$

b) KI för skillnad mellan två proportioner:

$$p_1^* = \frac{X}{n} = \frac{11}{100} \qquad p_2^* = \frac{Y}{n} = \frac{8}{100}$$

$$I_{\Delta p} = p_1^* - p_2^* \pm \lambda_{\alpha/2} \cdot \sqrt{\frac{p_1^*(1-p_1^*) + p_2^* \cdot (1-p_2^*)}{n}}$$

$\alpha = 0.01$

$\lambda_{0.005} = 2.58$

$d = 0.0414$

$$I_{\Delta p} = 0.03 \pm 2.58 \cdot 0.0414 = 0.03 \pm 0.1067 = (-0.077; 0.137)$$

c) nej, intervallet innehåller 0.

Tenta MS0065 HT17

8. t-test $\mu_0 = 20$ $s = 0.155$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{18.22 - 20}{\sqrt{0.024}/\sqrt{10}} = -36.33$$

$$\Omega_\alpha = \{ t < -t_\alpha(n-1) \}$$

$\alpha = 0.05$

$t_{0.05}(9) = 1.8331$

$t < -t_\alpha(n-1)$?

$-36.33 < -1.8331$ TRUE förkasta H_0

$\rightarrow$ accepterar $H_a : \mu < 20$

$\rightarrow$ Köp!

- några har räknat ut ett ensidigt KI:

$$I_\mu = \left(-\infty, \bar{x} + t_\alpha(n-1) \cdot \frac{s}{\sqrt{n}}\right)$$

$$= (-\infty, 18.31)$$ med samma slutsats!